data. The advantage of using genetic algorithm is that it is able to identify globally optimum results. It is found that, on average, the wavelet-updated model performs better than the FRF-updated model.

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Aeroelastic Response of an Airfoil-Aileron Combination with Freeplay in Aileron Hinge

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Introduction

RECENTLY, increasing attention has been devoted to nonlinear aspects of aeroelastic problems. One particular nonlinearity that has received considerable attention is bilinear structural stiffness. This is a good representation of either loose or worn control-surface hinges. Freeplay nonlinearity was first considered by Woolston et al.[1] Their results, both analog simulation and experimental, showed that depending on the initial pitch displacement limit-cycle oscillations (LCOs) can occur for velocities significantly less than the linear flutter velocity. Flutter analyses of airfoils with freeplay structural nonlinearities were then continued by several researchers, for example, Breitbach,[2] McIntosh et al.,[3] Tang and Dowell,[4] and Price et al.[5] These studies confirmed the existence of complicated dynamics well below the linear flutter velocity. Recently, Kholodar et al.[6] analyzed the behavior of a three-degree-of-freedom (DOF) airfoil via numerical time integration using a standard state-space approximation to Theodorsen aerodynamics, and they obtained a variety of nonlinear behaviors.

The objective of this Note is to show some interesting dynamical behaviors of a three-DOF airfoil-aileron combination subjected to incompressible airflow taking into account a freeplay structural nonlinearity in the aileron hinge moment. At first, because the airfoil motion can be nonperiodic, aerodynamic forces for arbitrary motions of a three-DOF airfoil are derived from Theodorsen's equations using Laplace transformation.[7] Then, the airfoil's dynamical behavior and possibility of nonperiodic motions are investigated using both a finite difference method and a dual-input describing function technique.[5] Results from both methods are presented here because the finite difference results do not show complete continuous form of the LCOs, and the describing function method is not able to give nonperiodic solutions.

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Aeroelastic Equations

The equations of motion for the two-dimensional airfoil-aileron combination shown schematically in Fig. 1a can be written in nondimensional form as

$$\xi''(\tau) + x_\alpha \alpha''(\tau) + x_\beta \beta''(\tau) + 2\zeta_\xi(\bar{\omega}_\xi/U)\xi'(\tau) + (\bar{\omega}_\xi/U)^2\xi = p(\tau, \xi, \alpha, \beta) \tag{1}$$

$$\left(x_\alpha/r_\alpha^2\right)\xi''(\tau) + \alpha''(\tau) + \left(z_\beta/r_\alpha^2\right)\beta''(\tau) + 2\zeta_\alpha(1/U)\alpha'(\tau) + (1/U^2)\alpha = r(\tau, \xi, \alpha, \beta) \tag{2}$$

$$\left(x_\beta/r_\beta^2\right)\xi''(\tau) + \left(z_\beta/r_\beta^2\right)\alpha''(\tau) + \beta''(\tau) + 2\zeta_\beta(\bar{\omega}_\beta/U)\beta'(\tau) + (\bar{\omega}_\beta/U)^2 H(\beta) = w(\tau, \xi, \alpha, \beta) \tag{3}$$

where $\xi = h/b$ is the nondimensional heave displacement; $(\,)'$ denotes differentiation with respect to nondimensional time $\tau = tV/b$; $H(\beta)$ is a nonlinear function representing the restoring moment in the aileron hinge normalized with respect to the linear stiffness; p, r, and w are the nondimensional aerodynamic force and moments defined as $p(\tau) = -L/(mV^2/b)$, $r(\tau) = -M_\alpha/mV^2r_\alpha^2$, and $w(\tau) = -M_\beta/mV^2r_\beta^2$; U is nondimensional airspeed; r_α is the nondimensional airfoil radius of gyration about the elastic axis, r_β is the nondimensional aileron radius of gyration about the aileron hinge line, and $z_\beta = r_\beta^2 + (c_\beta - a_h)x_\beta$; and ζ_ξ, ζ_α, and ζ_β are viscous damping ratios in plunge pitch and aileron, respectively. $\bar{\omega}_\xi$ and $\bar{\omega}_\beta$ are uncoupled frequency ratios defined as $\bar{\omega}_\xi = \sqrt{[(K_\xi/m)/(K_\alpha/I_\alpha)]}$ and $\bar{\omega}_\beta = \sqrt{[(K_\beta/m)/(K_\alpha/I_\alpha)]}$, where m is the mass of airfoil-aileron; I_α the mass moment of inertia of the airfoil-aileron about the elastic axis; I_β the mass moment of inertia of the aileron about the aileron hinge; and K_ξ, K_α, and K_β are linearized stiffnesses in plunge, pitch, and aileron hinge, respectively.

Because of the possibility of nonperiodic motions of the airfoil, Theodorsen's equations cannot be employed in the present analysis. Thus, the aerodynamic force and moments are derived for any

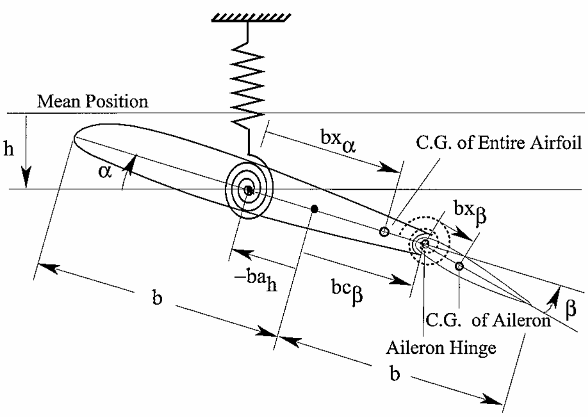


Fig. 1a Schematic of the three-DOF airfoil.

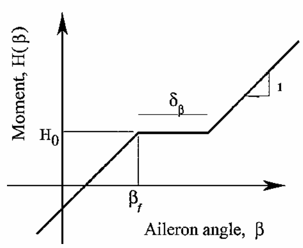


Fig. 1b Schematic of the freeplay nonlinearity.

arbitrary motion of the airfoil-aileron from Theodorsen's equations by means of a Fourier analysis,[7] giving

$$L(\tau) = \pi\rho b V^2[\xi''(\tau) - a_h\alpha''(\tau) - (T_1/\pi)\beta''(\tau) + \alpha'(\tau) - (T_4/\pi)\beta'(\tau) + 2(XTM)] \quad (4)$$

$$M_\alpha(\tau) = \pi\rho b^2 V^2\Big\{a_h\xi''(\tau) - \left(\tfrac{1}{8} + a_h^2\right)\alpha''(\tau) + (1/\pi)[T_7 + (c_\beta - a_h)T_1]\beta''(\tau) - \left(\tfrac{1}{2} - a_h\right)\alpha'(\tau) - (1/\pi)[T_1 - T_8 - (c_\beta - a_h)T_4 + T_{11}/2]\beta'(\tau) - (1/\pi)(T_4 + T_{10})\beta(\tau) + 2\left(\tfrac{1}{2} + a_h\right)(XTM)\Big\} \quad (5)$$

$$M_\beta(\tau) = \pi\rho b^2 V^2\Big\{(T_1/\pi)\xi''(\tau) + (1/\pi)[T_7 + (c_\beta - a_h)T_1]\alpha''(\tau) + \left(T_3/\pi^2\right)\beta''(\tau) + (1/\pi)\left[\tfrac{1}{3}\left(1 - c_\beta^2\right)^{\frac{3}{2}} + T_1 + T_4/2\right]\alpha'(\tau) + \left(T_4T_{11}/2\pi^2\right)\beta'(\tau) + \left[(T_4T_{10} - T_5)/\pi^2\right]\beta(\tau) - (T_{12}/\pi)(XTM)\Big\} \quad (6)$$

where

$$(XTM) = \phi(\tau)\left[\xi'(0) + \left(\frac{1}{2} - a_h\right)\alpha'(0) + \frac{T_{11}}{2\pi}\beta'(0) + \alpha(0) + \frac{T_{10}}{\pi}\beta(0)\right]\phi(\tau) + \int_0^\tau \phi(\tau - \sigma)\left[\xi''(\sigma) + \left(\frac{1}{2} - a_h\right)\alpha''(\sigma) + \frac{T_{11}}{2\pi}\beta''(\sigma) + \alpha'(\sigma) + \frac{T_{10}}{\pi}\beta'(\sigma)\right]d\sigma$$

Figure 1b shows the nonlinearity assumed for the aileron hinge moment. The nonlinear moment H_β is given by

$$H(\beta) = \begin{cases} \beta - \beta_f + H_0 & \text{for} \quad \beta < \beta_f \\ H_0 & \text{for} \quad \beta_f \le \beta \le \beta_f + \delta_\beta \\ \beta - \beta_f - \delta_\beta + H_0 & \text{for} \quad \beta + \delta_\beta < \beta \end{cases}$$

Results and Discussion

The results presented here are for the following airfoil, aileron, and nonlinearity parameters: $\bar{\omega}_\xi = 0.4$, $\bar{\omega}_\beta = 1.5$, $\mu = 100$, $a_h = -0.5$, $x_\alpha = 0.25$, $r_\alpha = 0.5$, $x_\beta = 0.002$, $r_\beta^2 = 0.002$, $H_0 = 0$, $\beta_f = -0.5$, and $\delta_\beta = 1$ deg. For this airfoil the linear flutter speed is found to be $U^* = 5.237$.

Figure 2 is a typical bifurcation diagram of the aileron response. Results obtained using both the finite difference and describing function methods are presented. These methods are explained in Ref. 5. The describing function results show, as a function of U/U^*, the amplitudes of both stable and unstable LCOs. The finite difference results show the value of α when $\alpha' = 0$ (that is, extremums of the steady-state oscillations). The significance of the finite difference results is as follows: if at a particular U/U^* the system is stable, then a single point is obtained, for example, $0.9 < U/U^* < 0.98$; if the motion is an LCO with one frequency, then two points are

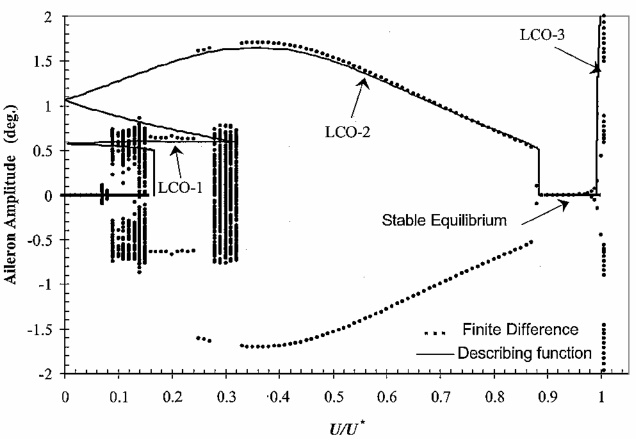
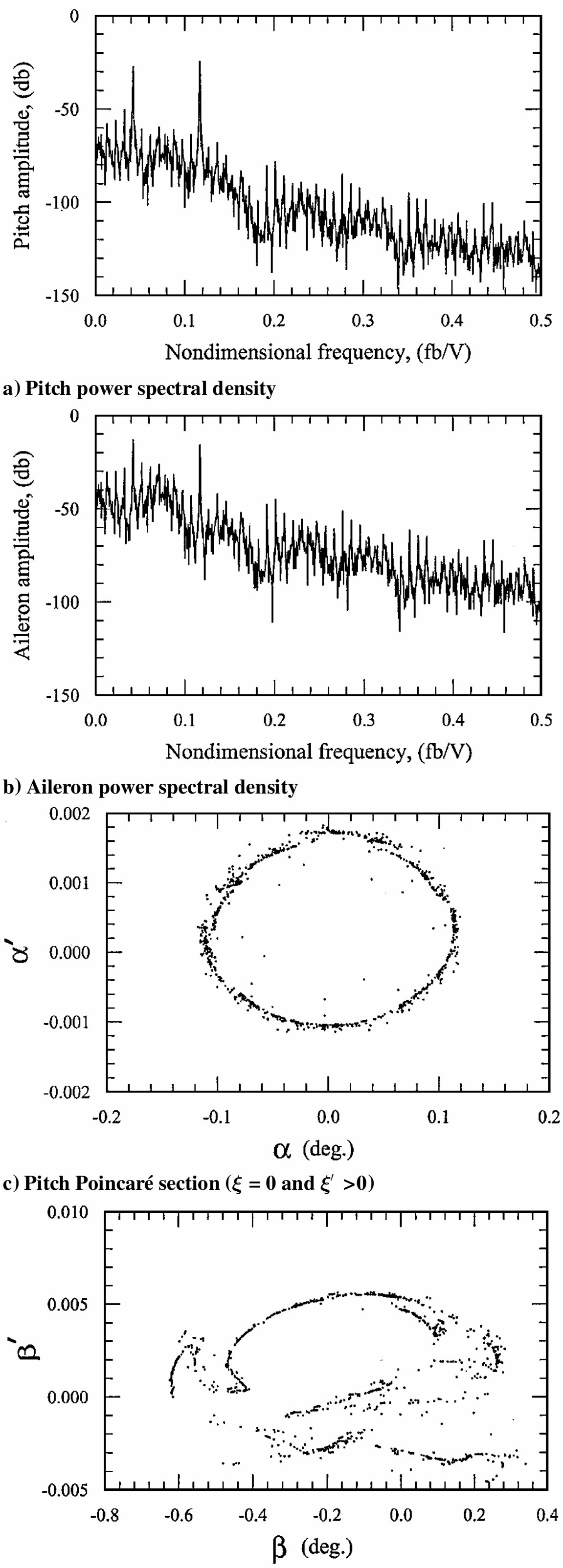


Fig. 2 Bifurcation diagram: $\bar{\omega}_\xi = 0.4, \bar{\omega}_\beta = 1.5, \mu = 100, a_h = -0.5, x_\alpha = 0.25, r_\alpha = 0.5, x_\beta = 0.002, r_\beta^2 = 0.002, H_0 = 0, \beta_f = -0.5$, and $\delta_\beta = 1$ deg.



a) Pitch power spectral density

b) Aileron power spectral density

c) Pitch Poincaré section ($\xi = 0$ and $\xi' > 0$)

d) Aileron Poincaré section ($\xi = 0$ and $\xi' > 0$)

Fig. 3 These views are for the same airfoil as Fig. 2 and $U/U^* = 0.3$.

obtained, for example, $0.4 < U/U^* < 0.9$; and, an LCO with two frequencies results in four points, etc. A large number of points at some velocities indicates aperiodic or possibly chaotic motion, for example, $0.27 < U/U^* < 0.33$. The finite difference results give only one stable solution for each given set of initial conditions and velocity, whereas the describing function method produces all possible stable or unstable solution regardless of the initial conditions.

As shown in Fig. 2, the system has three stable LCOs and one stable equilibrium solution below the linear flutter speed. In addition to the LCO solutions, the finite difference results show nonperiodic type motions for some airspeeds, most notably $0.27 < U/U^* < 0.33$. For a speed range $U/U^* < 0.32$, Fig. 2 indicates the possibility of two stable period-one LCOs. These oscillations are analyzed via power spectral density. Power spectra obtained for the two LCOs at each speed suggest that the frequency ratio of the two oscillations is most probably irrational. This indicates the possibility of quasi-periodic oscillations, which is a precursor to the quasi-periodic route leading to chaos.

For the given set of airfoil parameters and $U/U^* = 0.3$, phase-plane plots suggest nonperiodic oscillations in all pitch, plunge, and aileron motions. Figure 3 shows power spectral densities and Poincaré sections of both the pitch and aileron motions at $U/U^* = 0.3$. The broadband frequency spectrum (Fig. 3b) and fractal-like pattern in the Poincaré map (Fig. 3d) are strong indicators of chaotic motion for the aileron motion. For the pitch motion, however, the frequency spectrum is less broadband, and it shows broadening of two frequency spikes, possibly with an irrational frequency ratio (Figs. 3a). Also, the Poincaré map of the pitch motion is nearly a closed curve (Fig. 3c). These are characteristics of either a quasi-periodic oscillation or very mild chaos obtained through the quasi-periodic route. To classify the pitch motion more precisely, Lyapunov exponents should be calculated. The difference in dynamics of the pitch and aileron motion might be explained by the weak elastic coupling between the structural modes and the aileron deflection β [that is, $r_\beta = 0.002$ and $x_\beta = 0.002$; see Eqs. (1–3)]. Based on the preceding analysis and other frequency responses not presented here, it can be concluded that there is a quasi-periodic route to chaos for this system. This clarifies the airfoil's behavior below the flutter speed, which is useful for preventing nonlinear flutter. More results and analysis of the system behavior are presented in Ref 7.

Conclusions

To properly study the three-DOF airfoil's nonlinear dynamics, aerodynamic forces for arbitrary motions of the three-DOF airfoil are derived from Theodorsen's equations by means of a Fourier analysis. Behavior of the airfoil-aileron combination is then studied taking into account a freeplay nonlinearity in the aileron hinge moment. Bifurcation analysis of the system indicates various LCO solutions for velocities well below the linear flutter boundary. For some range of airspeed, there exists two LCOs with irrational frequency ratios giving rise to quasi-periodic oscillations. Depending on the initial conditions and airspeed, quasi-periodic and chaotic oscillations are also observed for the aileron motion. Thus, these results show that proper analysis of nonlinearities will lead to a better explanation of classes of aeroelastic responses.

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Camber Effects on the Near Wake of Oscillating Airfoils

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Nomenclature

C	=	airfoil chord
f	=	frequency of oscillation
K	=	reduced frequency, $K = \pi f C / U_\infty$
k_1, k_2	=	yaw factor
Re	=	Reynolds number
t	=	time
U_∞	=	freestream velocity
u	=	velocity component
u'	=	turbulent velocity fluctuation
X, Y	=	streamwise, normal coordinates
α	=	instantaneous angle of attack
α_0	=	mean incidence angle
α_1	=	oscillation amplitude

Introduction

THE study of the near-wake characteristics of an oscillating airfoil has significant scientific and engineering applications. The near flow behind a propeller and the near wake of a helicopter blade are perturbed by unsteady incoming flows. Both the mean velocity defect and the turbulence properties of the wakes play a significant role in the noise generation, inefficiency, and performance of the subsequence devices (for example, propeller, rotor blades, etc). Hence accurate prediction of the near-wake properties is necessary in the design of efficient airfoils.

Most of the previous research efforts in this area are directed to the phenomenon of the flow structure over the streamlined symmetric airfoils. Hah and Lakshminarayana[1] have studied the mean velocity and turbulence structure in the near wake of a symmetric airfoil (NACA 0012) experimentally and numerically. The results showed the complex nature of the near wake and its asymmetrical phenomena. Park et al.[2] investigated the characteristics of the near wakes behind a NACA 0012 airfoil at a given combination of mean

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